Teaching Tidbit: Multiplying the “New Way”

W. Blaine Dowler

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In my day job, I work in private education, spending most of my time supporting students who struggle in the public school system[1]. In the past few years, some local school boards have changed the way they teach multiplication, and the methods used are confusing a lot of students and parents. It’s becoming more difficult for parents to help their own children, as they are unfamiliar with the new methodologies.

1 The Impetus

This all started fairly innocently. The goal was to take one of school’s most difficult subjects (math) and find a way to make it easier so that students can be more successful with the same content. This is a highly admirable goal. While researching this, students who hadn’t learned topics like multiplication were prompted to solve multiplication problems based solely on the definition that multiplication is repeated addition. Researchers found that the students they were working with were intuitively trying to multiply, add, subtract, etc. from left to right rather than from right to left. The conclusion they came to was to find methodologies in which this left to right approach was actually used, so that the content would be more consistent with intuition.

2 The Old Method for Multiplication

As an example, let us take the problem 423×67. In the “old-fashioned” way, we start by multiplying the top row of digits (top number) by the 7 in the ones place of the second number, carrying any tens digits above the next highest digit, like so:

$$\begin{array}{r} {}^{1}4^{2}23 \\ \times 67 \\ \hline 2961 \\ \\ \hline \end{array}$$

[1]No, I won’t say exactly where I work here. This document does not represent the opinions of my employer. The entity which employs me has, to the best of my knowledge, no official opinion of any kind about this trend. The opinions expressed in this document are mine and mine alone.

In the second step, we add a zero to the end of the second line to account for the fact that the 6 in our multiplier represents a 6 in the “tens” place, and get

$$\begin{array}{r} 423 \\ \times 67 \\ \hline 2961 \\ 0 \\ \hline \end{array}$$

Now we multiply by the 6, and get

$$\begin{array}{r} {}^{1}4{}^{1}23 \\ \times 67 \\ \hline 2961 \\ 25380 \\ \hline \end{array}$$

Adding the bottom two rows, carrying to the next digit left when needed, gets us the final answer:

$$\begin{array}{r} {}^{1}4{}^{2}23 \\ \times 67 \\ \hline 2961 \\ 25380 \\ \hline 28341 \end{array}$$

3 The New Method for Multiplication

The new method changes the way we carry, or regroup, our numbers. It is also driven by working from left to right. We use the same example:

$$\begin{array}{r} 423 \\ \times 67 \\ \hline \end{array}$$

This time around, however, we start by multiplying the 4 by the 6, which produces 24. We write

$$\begin{array}{r} 423 \\ \times 67 \\ \hline 24 \end{array}$$

Note that the 24 is aligned one place value to the left of the ”423” on the top line. Next, we multiply the 2 by the 6 and get 12. We can write the 2 after the 24 easily enough, but what do we do with the 1? We could add it to the 4, but the new methodologies do not recommend crossing digits off. So, we write the 2 after the 24 and the 1 beneath the 4, as follows:

$$\begin{array}{r} 423 \\ \times 67 \\ \hline 242 \\ 1 \end{array}$$

We continue similarly when multiplying the 3 by the 6, finally adding the placeholder 0 at the end of the line. In this case, we just add zeroes until all place values have been filled, meaning we don't need to count the number of 0 digits we need to add here, because we did that when positioning the 24 in the first step. We do the same in the second row.

$$\begin{array}{r} 423 \\ \times 67 \\ \hline 24280 \\ 1100 \end{array}$$

Now we add our last two rows to obtain

$$\begin{array}{r} 423 \\ \times 67 \\ \hline 24280 \\ 1100 \\ \hline 25380 \end{array}$$

Our next step is to multiply by the 7. This has no digits after it, so when we calculate $4 \times 7 = 28$, we align the 8 directly under the 4:

$$\begin{array}{r} 423 \\ \times 67 \\ \hline 24280 \\ 1100 \\ \hline 25380 \\ 28 \end{array}$$

Now we continue to multiply, carrying beneath the number again, and obtain

$$\begin{array}{r} 423 \\ \times 67 \\ \hline 24280 \\ 1100 \\ \hline 25380 \\ 2841 \\ 120 \\ \hline \end{array}$$

We now add up the *three* numbers below, carrying to yet another new row, to get

$$\begin{array}{r}
423 \\
\times 67 \\
\hline
24280 \\
1100 \\
\hline
25380 \\
2841 \\
120 \\
\hline
27241 \\
1100 \\
\hline
28341
\end{array}$$

If multiplying by a number with more digits, we need to count out the placeholders and add them up. So, where the old fashioned methods would have us solve 987×654 as

$$\begin{array}{r}
987 \\
\times 654 \\
\hline
3948 \\
49350 \\
592200 \\
\hline
645498
\end{array}$$

the new methods have us solve it as

$$\begin{array}{r}
987 \\
\times 654 \\
\hline
548200 \\
44000 \\
\hline
582200 \\
10000 \\
\hline
592200 \\
45050 \\
4300 \\
\hline
531550 \\
110000 \\
\hline
641550 \\
3628 \\
320 \\
\hline
644498 \\
1000 \\
\hline
645498
\end{array}$$

Gee, I sure am glad they found a way to simplify this whole multiplication mess for us by going from left to right and not crossing things out anymore.

4 My Personal Critique

I should start this section by noting that I am not an education researcher. I did some research in physics, but my formal education background is a B.Ed. Therefore, it is entirely possible that the people with Ph.D.s in the subject have seen advantages I'm missing here. This is what I do know:

1. Our number system is the Arabic number system. The Arabic language reads from right to left. The reason most North American and European students try to approach things left to right is that they have been trained this way by their language, not by the human genetic code. Math works right to left. The further you get, the more important algebra will become. By the time students finish High School, they will have seen topics in math that *absolutely require* working from right to left. The professional mathematical world isn't about to change.

2. This was tried for the same reasons in the late 1800s. In just under 30 years, it was abandoned as a catastrophic failure. When I've spoken to people involved in driving the movement forward at a local level, asking them what they've done differently to prevent the failures of the previous attempts, the answer (now received from six different individuals over the course of three years) has been the same every time: "this has been tried before?"

3. Anecdotal evidence is showing a lot of students who are making errors in multiplication because they are misaligning the placeholder zeroes, or forgetting to put them in at all because their natural instincts are to put their answer to multiplication directly below the numbers they are multiplying. They then run out of space for the placeholder zeroes, and leave them out. This gets worse with carrying below the row of numbers. Take 1305×72 for example. It is not at all uncommon for me to see students write

$$\begin{array}{r} 1305 \\ \times 72 \\ \hline 7105 \\ 23 \\ \hline 7128 \end{array}$$

   instead of

$$\begin{array}{r} 1305 \\ \times 72 \\ \hline 7105 \\ 2030 \\ \hline 9135 \end{array}$$

   with the predictable results.

4. This evidence is not anecdotal, but specifics cannot be shared. As a part of my day job, I have access to enrollment numbers for a region which is

home to several hundred thousand school age students. NDAs prevent me from sharing specific numbers. However, the usual trend over the past 20+ years has been for enrollments to spike the first year a new curriculum is implemented, and then drop off as teachers and textbook writers identify and correct issues with instruction. These spike are typically in the 5-7% range for the first year, before returning to previous levels within two years. Regions using this method in their new curriculum are seeing 10-15% spikes in the first year, followed by 2-5% increases each year thereafter until attendance levels stabilize about 20% higher than they originally were. I find this to be very telling.

5 Conclusion

I hope this will help parents facing this curriculum learn this method of multiplication to help their children. I have no kids of my own just yet, but I'll be setting aside time to work with my nieces outside of school when they hit these grades.