Electricity and Magnetism
Version B: Mathless

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April 21, 2012

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1 Electricity and Magnetism the Newtonian Way

We have already seen how the laws of electricity and magnetism led to the rise of relativity. Let us examine how relativity alters them more closely.

Both electrical and magnetic phenomena have been studied for thousands of years. It was not until 1820, however, that the scientific community at large recognized how closely connected the two are. Hans Christian Oersted noticed (which giving a lecture to students) that electric currents deflect the needle of a compass. Further studies were able to quantify the relationship, explicitly defining and mathematically defining the procedure. It was clear that an electrically charged object in motion produces a magnetic field that is circular in shape and surrounds the particle at a right angle to the direction of motion. There was a problem, though: there was no *theoretical* reason to connect the two interactions. The two types of phenomena were kludged together mathematically because the connection was experimentally proven, and *not* because of any intrinsic motivation to tie the two together. It was comparable to rain and rainbows: people recognized that rainbows appeared after rainfalls so early on that even the word “rainbow” includes the word “rain,” but it would be centuries before the optics involved were understood well enough to explain precisely why rainbows appeared.

The theoretical descriptions of electrical and magnetic phenomena was completed in the late 1860s, and James Clerk Maxwell collated the results into what are now known as Maxwell’s equations. These described all known electrical and magnetic phenomena, detailing the connections between the fields without explaining why they are connected, with particular focus and attention on how the electric and magnetic fields change and evolve through space and time.[1] Just as ancient people could predict the appearance of rainbows, people in the 1900s could predict the magnetic fields produced by electrically charged objects to great effect, but the connection was arbitrary. This was still a young science, so when it was shown to be inconsistent with Newtonian physics, it was assumed that the fault lay with electricity and magnetism. As both theories were consistent with the experimental data available at the time, this assumption seemed reasonable. As we learned in earlier lessons, it was actually the Newtonian mechanics that were at fault.

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[1] Remember, to James Clerk Maxwell and his contemporaries, space and time were two completely distinct quantities.

2 Electricity and Magnetism the Relativistic Way

The work of Lorentz and Einstein combined to solve the problems with electricity and magnetism in multiple frames of reference. Furthermore, they showed that electromagnetic theory did not need to be altered in any special way; the framework established by Maxwell's equations was completely consistent with electricity and magnetism. There were, however, other significant benefits to the relativistic framework that were worth exploring.

So far, when we take a relativistic viewpoint of quantities, we have found a common pattern: quantities that depend on direction get a fourth quantity attached which serves as the "time direction" component of the quantity. In all such cases, the fourth quantity was independent of direction. In the case of electrical fields, the prime candidate for such a fourth quantity is the magnetic field, but that *also* depends on direction. It can't fit easily into that fourth slot. This seems to be a stumbling block on the path to connecting electricity and magnetism on the theoretical side.

Undeterred, we go back to what we learned in lesson four: although no *simple* rotation can move something from a spatial direction into the time direction, one can formulate a rotation based on acceleration that does the same job. This becomes the key to connecting the two phenomena.

Imagine, if you will, a long, electrically charge rod that is at rest. In the classical view, accelerating this rod along its axis[2] causes no change in the electrical field, but produces a sudden and inexplicable magnetic field. If we had two such rods lying parallel and moving in identical directions at identical speeds, the magnetic force created would be an attractive force.

We now know that such a motion will increase the electrical force: as the rods accelerate, their lengths contract. With the same amount of charge packed into a shorter distance, we see the electrical charge density increase, which causes an increase in the electrostatic attraction. By carefully calculating the result of "rotating" the electric field to a higher velocity using the math that describes our relativistic world, we find that the action of the classical magnetic field is *identical* to the action of this "extra" electric field produced by relativity! This math is independent of the situation, so it doesn't matter if our moving electrical charge is a rod, disc, point, etc. This is true for *any* shape that moves while carrying an electrical charge.

This was something of a holy grail to physicists: the elusive theoretical connection between the electric and magnetic fields that didn't exist in the classical

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[2] By "along its axis," we mean that if the rod lies in the North/South direction, then we accelerate it either North or South, and not East, West, up or down.

view became an immediate and natural consequence of relativistic dynamics. The magnetic fields observed around active circuits resulted directly from the motion of charged particles under the rules of relativity. The mathematical object used to combine the two is more complicated than a four directional vector, but it is just as valid: the fields were unified into a single phenomenon, and Maxwell's set of four[3] equations simplified to a single equation. This served as yet another triumph of the special theory of relativity. Despite a rocky start, the scientific community had finally accepted special relativity as the theory that explained reality better than any other theory. It is now known that any errors in relativity are small, as the theory has been tested with incredible experimental rigor. Still, special relativity is limited to points of view that do not experience forces or accelerations. Most of our work is done in the presence of a gravitational force, and many real world observers need to change their direction to do their jobs. The special theory of relativity was inadequate for these tasks, and would need to be expanded into the general theory of relativity, which is the subject of our final lessons.

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[3] Well, technically, Maxwell did his work before physicists adopted vectors as mathematical tools, so his "four" equations were originally published as twelve equations. In a roundabout sort of way, this also explains why the symbols for so many electromagnetic variables are counter intuitive.