

Energy and Momentum

Version B: Mathless

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1 What Is Energy?

One of the quantities that physicists find most useful is energy. It is often difficult to define for new students, due in large part to the fact that it is an intangible quantity that you cannot see, touch, taste or smell. The formal definition is often “energy is the ability to do work,” which seems clear and straightforward until one is asked to define “work” in the scientific sense.

An alternative definition of energy is “the ability to cause changes in motion or position.” From a scientific perspective, this is equivalent to the first definition, but it reduces the terminology to that which is intuitive for new students. There are different types of energy in the world, though we will only deal with two of them in this series.

1.1 Kinetic Energy

The type of energy that is often the easiest for students to accept is kinetic energy, or energy of motion. If an object is in motion, its own position is constantly changing. It also has the potential to cause changes in motion to other bodies through collisions.

In the Newtonian view, kinetic energy can be calculated based solely on the mass and speed of an object. If two objects with different mass travel at the same speed, then the object with greater mass has greater kinetic energy. If two objects with the same mass travel at different speeds, then the one with the greater speed has the greater kinetic energy. Moreover, the speed of the object has significantly greater impact than the mass. (If you double an object’s mass without changing its speed, you double its kinetic energy. If you double its speed, you more than double its kinetic energy.)

1.2 Potential Energy

The second major type of energy is potential energy. This is a little harder to see, as there is nothing visibly moving or altered in any way as a result of obtaining potential energy. This energy can be “released” and turned into kinetic energy in one form or another. For example, a book on a shelf has gravitational potential energy: the book might fall, gaining speed and kinetic energy as it approaches the floor. A book on a lower shelf has less potential energy, as it cannot fall as far and will be traveling at a lower speed when it hits the ground.

Potential energy is a consideration when there is some sort of outside field or force that can drive changes in motion, such as gravity. As such, further discussion will be saved for the general theory of relativity. The variables one needs to describe potential energy depend upon the source and nature of the outside force or field.

2 What Is Momentum?

Momentum is a term that is commonly used in day to day speech in a fashion that differs from its scientific meaning. When people talk about momentum in the colloquial sense, they typically mean something is in motion and hard to stop. This usage is closer to the word “inertia” than momentum. It is inertia that determines the resistance to accelerations and changes in motion. Momentum, in the Newtonian view, is the product of mass and velocity.¹ In Newton’s day, it was referred to as the “quantity of motion.” Momentum is always *conserved* in a collision, meaning that the total momentum that goes into a collision is the same as the total momentum that comes after. (Again, direction matters; two vehicles meeting head on can turn into a stationary wreck immediately after collision because momentum in one direction, say North, can be canceled out by momentum in the opposite direction, South.)

3 How Are They Connected?

Kinetic energy and momentum can both be calculated using the same two pieces of information: the mass of the body in motion, and the velocity of the body in motion. Although they are not identical quantities,² the fact that they depend solely on the same two variables begs the question about whether or not the two are related.

3.1 The Newtonian View

In the Newtonian view, energy and momentum are connected through forces. When a force is applied to a object at rest, it accelerates to some velocity. The force applies over a period of time, during which the object travels some distance. Assuming the object is in space, so we don’t have to worry about friction and

¹It uses velocity, and not speed, because direction matters.

²They cannot be the same: direction matters to momentum but not kinetic energy, which is a significant difference on the conceptual side.

other factors, allowing the entire force to apply to accelerating the object in the exact direction of the force, then we can calculate both the energy and momentum using simple and similar math. We will also assume that the force is uniform, meaning it applies with the same strength and in the same direction the entire time it is applied. To calculate the energy of the object, one multiplies the force by the distance it has traveled. To calculate the momentum of the object, one multiplies the force by the time elapsed during the acceleration.³

3.2 The Relativistic View

In the relativistic view, there are a few changes that need to be made to these concepts. The first and foremost idea that needs to be explored is whether or not there is an upper limit to either quantity. After all, the universe has a speed limit in the speed of light. If these quantities depend on the speed of an object, then one would tend to think that there could be an upper limit to either or both in turn.

The relationship between the two quantities, as force applied along different directions (space or time), still applies in relativity. In that case, conceptually, we see that there should be no upper limit to either quantity: one can continually apply a force to an object, regardless of how much time elapses or distance is traveled. So, how do we obtain unlimited energy and momentum with a limited speed?

The answer lies in the fact that we have misidentified the variables involved. As we mentioned in lesson two, Newton originally recognized inertia as a key quantity, and then proposed that the mass and inertia of an object were one and the same quantity. Although he couldn't possibly have realized this at the time, mass and inertia are not, in fact, the same. The inertia of an object increases as its speed increases. This increase is almost imperceptible at the speeds that we see in our everyday experiences, but it is unmistakable at higher speeds. As an object gets closer to the speed of light, its inertia increases incredibly rapidly, ensuring that no amount of energy or momentum will carry the body above the speed of light. In several of the cases in which we used mass in Newtonian mechanics, we should have used inertia as Newton originally proposed. This concept was so counter intuitive, and the notion of equality between mass and inertia so widely accepted, that when this difficulty was first noticed physicists incorrectly assumed that the mass of an object was increasing, and the quantities were named as such. The mass was referred to as *rest mass* and the inertia was referred to as *relativistic mass*. Although the error in these terms is now understood, they were in use for so long that they still appear in much of the literature on the subject.

³If the force is not uniform, some adjustments need to be made. Instead of simple multiplication, we use calculus to integrate the force with respect to the other variable.

The next question pertains primarily to momentum: with momentum, direction matters, so what quantity appears as the time component of momentum? We now have four directions that need to be described when dealing with quantities that use direction at all. To discover this component, we begin with Newton's definition of momentum as the product of inertia and velocity. We take our four dimensional velocity and multiply it by the inertia of an object. What we find in the time component of the result is startling: it is the kinetic energy of the object, with an additional term added: mc^2 , where m is the mass of the object and c is the speed of light.

This was utterly shocking. An object at rest contained energy in the absence of outside fields or forces, and that energy E had the quantity mc^2 . Never before had it occurred to anyone that mass could somehow store energy, and yet it seemed apparent that mass was some form of condensed, solidified energy. This revelation is the one that led to nuclear power. It was so unexpected that Einstein himself discussed it tangentially in his paper, deviating from the calculation he worked on to make a special point of describing it. It is also one of the primary examples of serendipitous discovery in the sciences: who would have believed that nuclear energy would have been discovered by examining momentum in a universe with a speed limit?