The Postulate of General Relativity
Version B: Mathless

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1 Terminology: Special and General

What makes "special relativity" special? What makes "general relativity" general? These are common questions when one first researches the subjects, particularly when one is coming from a non-mathematical background.

In mathematical situations, research is often devoted to special and general cases. A "special" case is one in which specific values are known, or specific restraints are applied. For example, most elementary school math deals with special cases. Students solve questions such as 234×123 instead of solving the general case of $a \times b = c$ and then substituting $a = 234$ and $b = 123$ before solving for c.[1] Science works in a similar way. Special relativity considers the special case in which the reference frames one uses to measure from are experiencing uniform, or unaccelerated, motion. General relativity expands the theory to allow observers to move with arbitrary accelerations.

2 The Postulate of General Relativity

The distinction between special and general relativity boils down to a single realization: one cannot distinguish between different accelerations by feel alone. Imagine being placed in a sealed and soundproof room (with ample oxygen supply.) Imagine also that you can distinguish between "up" and "down" as directions. There is a particular side of this room that you can call a "floor." Einstein realized[2] that there is no physical sensation that can be used to distinguish between standing in a sealed room on the surface of a planet and standing in a sealed room with a rocket engine attached that forces the room to accelerate.[3] This equivalence led to the formulation of *non-inertial* reference frames, or accelerated reference frames.

[1] It is the author's opinion that the lack of explicit instruction about the usefulness and need for general cases rather than exclusively special cases is one of the main reasons students struggle with algebra. When faced with a massive paradigm shift in mathematical thinking that is not *treated* as a massive paradigm shift for fear of scaring students away, the students become convinced that algebra is an exercise in futility and harder than it needs to be. When the author taught algebra to first time learners, he started with lessons about the paradigm shift and the axioms of algebra and the need for special cases before getting into the nitty gritty steps. He then gave out a homework assignment asking students to manipulate high school physics formulae in the general case only, and the class average was 87%. When he later taught high school physics to a completely different group of students and gave those students the identical assignment, the class average was 63%. End rant.

[2] He was riding an elevator at the time.

[3] We are forced to assume that any noises or vibrations caused by the rocket engine are not transmitted to the room.

3 Gravity

Beyond allowing the formulation of non-inertial reference frames, this postulate connected gravity to reference frames, which were already connected to the speed of light. This was a monumental connection. At the time, only three forces were recognized in science: there was the electrical force, the magnetic force and the gravitational force. With the first two bound by relativity, scientists began searching for a unified theory that could bind the “final” force to the other two.[4] Developing this “Grand Unified Theory,” or GUT, became Einstein’s unrealized goal in life. Over 50 years after his death, such a theory has still not been formulated to the satisfaction of most physicists.

This formulation did, however, spawn a new revelation: mass and energy do not merely exist “in” the geometry of reality, they alter and define the geometry of reality. Imagine you are still in your earliest school years, and have been tasked with drawing a triangle. The implications of Einstein’s postulate are akin to watching your piece of paper change and transform in shape and size as you draw the triangle on its surface. A flat piece of paper ceases to be flat as soon as something is drawn on it, instead curving in a way that attracts all other shapes on the page to the one you have just drawn. It is this revelation that has led to the notion of the “fabric” of spacetime. The mental image conjured by the mathematics renders reality as a great sheet. Massive and/or energetic objects dent this sheet, altering the “fabric” of the sheet of reality.

4 The Geometry of Reality

This, once again, changed the way the geometry of the world was laid out. “Rotating” objects to different speeds caused ripples in the sheet, moving and shifting the dents in what we perceive as gravity. This even led to the theory of gravitational waves, which emanate from a source of gravity and propagate throughout the universe.

These dents and ripples have a great impact on the motion of objects. It is no longer natural to think of objects thinking in straight lines. They can move in the straightest possible lines, but that isn’t quite the same thing; one can drive a “straight” stretch of road for several kilometers, but odds are the road curves along the surface of the Earth as elevations change on the oblate spheroid

[4] By today’s count, there are four forces: the gravitational force, the electromagnetic force, the weak nuclear force and the strong nuclear force. Because electricity and magnetism are already connected and unified in our day to day situations, they are now treated as a single force. They have been unified with the weak and strong nuclear forces as well, but this unification is only visible under the extreme conditions created within particle accelerators, and so they are still treated as distinct forces in most circumstances.

we call home. We now talk about "geodesics," which are the straightest possible lines one can follow on a curved surface.

To thoroughly drive the strangeness of this concept home, we will look at a shape that cannot exist on a flat surface, but which every reader is likely familiar with without even realizing that this is the case. The shape we are talking about is a "biangle." This shape is formed when two lines are drawn as straight as possible (i.e. two geodesics) and meet in exactly two places. In the geometry taught in public school, this cannot happen, but the geometry taught in public school is restricted to the "special case" of flat surfaces. The "general case" of geometry allows this shape to exist. You will probably not find examples in your mathematics classroom, but are extremely likely to find them in your geography classroom.

Take a globe of the Earth. It is covered with lines of longitude and latitude. Some, but not all, of these lines are geodesics. We need to determine which are which. Imagine you have a globe from the factory before it has been painted or marked in any way. It is a perfect sphere. You take a marker and start drawing the straightest possible line that you can from any point on the surface. You will eventually return to the spot you started from.[5] As the globe started with no distinguishing features, all such geodesic lines would be identical when drawn carefully and accurately. This is the case with lines of longitude on a globe, and *not* most[6] lines of latitude. If one takes two lines of longitude for examples on the globe, you will find that they form a biangle; they meet at the north pole, continue along the straightest possible paths until they are parallel while crossing the equator, and then converge to meet at the south pole and complete the biangle.

Similarly, "rules" for shapes that we are aware of do not necessarily apply. For example, we are taught that all triangles have angles which add up to 180°. While this is absolutely true on a flat surface, one can easily form a triangle with three right angles on an appropriately curved surface. Begin at any point on the equator. Travel directly to the north (or south) pole. Make a 90° turn and continue until you reach the equator once more. Make another 90° turn (in either direction, East or West) and return to your starting point. The triangle mapped out by these three geodesics will have three right angles, for a total angle of 270°.

We can take this same issue one step further and demonstrate the "parallel transport problem." Again, imagine you are standing on the equator, facing North with your right arm held out in the Eastward direction. Walk to the north pole, and with each step, you keep your arm held out parallel to the

[5] Note that the author assumes your penmanship and artistic skills are significantly better than his own. He'd miss the target by a significant amount.

[6] There is a grand total of one line of latitude that forms a geodesic. Deducing which line that is indicates a good understanding of the topic.

direction is was in when you took your previous step. Upon reaching the north pole, turn right, keeping your arm parallel to its original direction so that it now points directly ahead of you. Walk until you reach the equator and turn right again; you will now either have your right arm across the front of your body, or you will extend your left arm to maintain the same direction.[7] Walk back to your starting point. Your extended arm has been held parallel to its previous direction every step of the way, and yet the arm that started out pointing East is now pointing South! This is the parallel transport problem: you cannot transport something pointed a particular direction and guarantee that its direction will remain constant across a curved surface. Those paths which keep their own direction constant (i.e. those that don't "feel" like turning to a walking person: if you point your arm straight ahead with your first step, it points straight ahead of you at every step; if you return to your original position, you are facing in your original direction when you do) are the geodesics for that surface.

The concept of the geodesic is not just one that seems correct, but is one that turns out to be fundamental to the behavior of objects in our universe. If we go back to Newton's original axioms, he stated "Unless an outside force is applied, a stationary object (object "at rest") will remain stationary and a moving object will continue moving at the same speed and in the same direction." In short, things move in straight lines. We now modify this to say that, in the absence of outside forces, objects move along geodesics.

This counter intuitive geometry had been explored by abstract mathematicians for centuries before its application to reality through relativity. With this toolkit available, physicists soon discovered theoretical objects that still hold imaginations enthralled to this day: black holes and worm holes.

[7] Your right arm is probably pretty sore at this point.