Curiouser and Curiouser

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1 Unanswered Questions

Our discussions in the previous lesson revealed a few problems with classical thinking. To summarize:

1. The basic building blocks of matter, called elementary particles, must all have zero volume. What, then, prevents them from piling in so closely together that the matter they form does *not* have zero volume?
2. How does a nucleus hold itself together, if the electromagnetic force trying to push the protons apart is so much stronger than gravity, and gravity is the only attractive force we know of that applies?
3. Why aren't electrons ever found in a nucleus?
4. Why is the mass of a nucleus different from (and often lower than) the total mass of its component particles?
5. Why does the mass of a nucleus change when it emits energy, even though that energy has no mass?
6. How does information about particle positions get exchanged between particles? That would seem to violate energy conservation.
7. How are new particles created?

2 Energy and Mass

The fifth question on the above list is actually the easiest to answer, not because the answer is intuitive in any way, but because the behaviour is consistent, reproducible, and involves relatively few variables.

Before: we have a nucleus of a given mass.

During: the nucleus spontaneously discharges energy.

After: the nucleus left has the same number of protons, same number of neutrons, and less mass.

The conclusion seems inescapable: it is possible to convert mass into energy. In fact, some more detailed experiments have revealed that the amount of mass "lost" is very closely related to the amount of energy discharged. Specifically, if the mass of the nucleus is reduced by mass m, and if the energy discharged is E, then the relationship between the two is given by the equation $E = mc^2$, where $c = 299,792,458\ m/s$, which is the speed of light in a vacuum. This equation is most closely associated with Einstein's relativity, because Einstein reached the same conclusion years before quantum physics reached that point, though he approached it from an entirely different direction.

3 Mass of a Nucleus: $2 + 2 = 3.9$?

As discussed in lesson one, a nucleus made of a given number of protons and neutrons generally has a mass less than the total masses of the protons and neutrons it is made of. In fact, every stable nucleus (i.e. any nucleus that does not spontaneously fall apart) is guaranteed to have a mass less than the total masses of the parts it is made of. How can this be? Why is the mass not only different, but *less* than its components? The first step towards finding the solution to this issue is by examining how we are measuring the mass of a nucleus in the first place.

We have yet to develop the kitchen scale that can detect the presence of a single nucleus. The traditional means of determining mass (put it on a scale, find the weight[1] and convert) is simply not an option for objects this small and hard to count. Instead, we have to study the way nuclei move and deduce the mass from this process. Most commonly, physicists would strip a single electron off a set of atoms to make sure they had an overall electric charge,[2] and then launch the atoms through a magnetic field. When a charged particle moves through a magnetic field, it moves in a circular arc. After measuring the radius of the arc produced with a known charge and known magnetic field, the "mass" of the nucleus can then be measured.

[1]Remember, mass and weight are different quantities. Mass is the amount of stuff something is made of, while weight is the force of gravity acting on that much mass. You will still have mass in a weightless environment, and the fastest way to lose weight is to move to a planet with weaker forces of gravity.

[2]In this case, the electric charge is that of a single, unbalanced proton.

The word "mass" in the previous paragraph appears in quotations for a reason. Newton made a mistake.[3] When studying an object in motion, we do *not* measure the mass of that object.

3.1 Mass and Inertia

When Newton first proposed his laws of motion, he made a distinction between two quantities that was ignored almost immediately.

- **Mass**: Mass is the amount of matter an object is made of.
- **Inertia**: Inertia is an object's resistance to changes in motion.

By all experiments available to Newton at the time he formed his theories, mass and inertia appeared to be one and the same. They are not.

We have already seen that energy and mass are closely connected. It stands to reason that, if energy is closely tied to mass, and mass is closely tied to inertia, then energy and inertia would also have a connection. That is, in fact, the case.

What we have been referring to up to this point as "mass" is truly inertia. The mass of a nucleus is *not* different from the mass of the particles it is composed of. Rather, the *inertia*, which is a combination of both mass and the energy required to glue that mass together, is different from the inertia of the particles it is composed of.

The first step to the solution has been taken: we realized that the common vernacular includes invalid assumptions dating back centuries.[4] In fact, the terminology is so ingrained that it is still common to refer to inertia as "mass" and the actual mass as "rest mass," meaning the "mass" we measure for a particle when it is not moving. These common conventions will be held for the rest of these lessons: "mass" will refer to inertia, and "rest mass" will refer to mass.

3.2 Not So Elementary, My Dear Watson

We can now explain why we have different values for the mass of a nucleus and the sum of the masses of its components. What we cannot explain is why the

[3] To his credit, he couldn't have possibly known any better at the time.

[4] This is *not* the last time this is going to happen through these lessons. Or the last time on this page, for that matter.

mass is reduced when we put a nucleus together.[5]

By what we've seen so far, we would expect the mass of a nucleus to be *greater* than the total mass of the particles we make it from. It takes energy to stick two particles together, so we should have the mass of those two particles combined, plus the energy (called *binding energy*) that holds them together.

Again, we find the solution is logical but not intuitive. The simplest explanation is this: protons and neutrons are not elementary, indivisible particles. Rather, they are also composed of other constituent particles.[6] This allows the mass of a combination of these particles to be less than the mass of the particles on their own. Through whatever mechanism they use to bind themselves together, it must take less energy to hold a proton's component particles together when it is in a nucleus than it does in isolation. The same is true for neutrons.

4 Radioactive Decay

We have already mentioned that some nuclei are stable, while others are not. The stable ones, when left undisturbed, do not change their fundamental nature over time. This is not true of all nuclei.

Some nuclei are unstable. Given time, they will transform into other nuclei, generally emitting other particles in the process. One common example is an isotope of carbon known as carbon-14. Isotopes are numbered by the grand total number of protons and neutrons in the nucleus; as all carbon nuclei have 6 protons, this nucleus also has 8 neutrons. Left alone for a sufficiently long amount of time, this nucleus will "decay," transforming into a nucleus of nitrogen-14, emitting other particles in the process. As with all natural decay processes, the total mass of the ejected particles is less than the mass of the original carbon-14 nucleus. This is the process scientists use for carbon dating, which they use to determine the age of archeological artifacts. (More on this later.)

The process of radioactive decay is better understood today than it was a century ago. The conservation of energy can be applied to explain why the mass always reduces in natural, spontaneous processes: the total amount of energy in your "ingredients" when you start limits the total mass and energy of the particles that result.

[5]The difference between the mass of a nucleus and the sum of the masses of its component particles is called the *mass defect*.

[6]Specifically, each proton and neutron is composed primarily of three quarks.

4.1 Muons and Taus and Neutrinos, Oh My!

Early experimenters took the opposite approach to decay; if you can *add* energy to a system, then perhaps you can create more particles. This was tried, by accelerating a particle with electric charge to a high kinetic energy, and then slamming it head-on[7] into another particle. Reactions were mixed. Along with the elation of finding out that the theory worked, and new particles could, indeed be created in this fashion, came the general confusion that resulted by creating particles never before identified by man.

One of the first of these particles to be created was the muon, named for the Greek letter mu (μ) that was arbitrarily assigned to it. In almost all respects, it appeared to be identical to the electron. It has considerably greater mass[8] and is unstable; given time, it will decay, leaving behind an electron and, when first observed, some massively confused researchers. Physicists have long held to the notions of energy and momentum conservation. Both conservation laws are logical and intuitive, and were consistent with centuries of observation. The decay of the muon was the first time these conservation laws were seriously questioned. When the muon decayed, the only observable output was the electron. Let's break down what that means.

Let's say we have a stationary muon; it has no kinetic energy whatsoever.[9] After some time, it decays. The only particle we observe coming out of the decay is an electron, which has significantly lower mass. Thus, if we conserve energy, the electron must be produced in motion; the kinetic energy the electron gains must match the difference in masses times the speed of light squared, by $E = mc^2$. This leads to another problem, however. Momentum[10] would not be conserved; the muon was at rest, but the electron is not. If the electron comes out stationary, then momentum would be conserved, but energy would not. Another observation seems, at first, to make things even worse: the electron ejected by the decay does not, in fact, account for all of the energy of the original muon. The electron's total energy is invariably less than the muon's.

We have a few options to reconcile these issues. One is to throw out one of the conservation laws. They are, after all, arbitrary human rules that have stuck around only because they have worked so incredibly well for centuries. Another is to propose yet another new particle, and a rather exotic one at that. If there

[7] This is merely a figure of speech. To the best of our knowledge, particles with zero volume do not have heads.

[8] The mass of the muon is over 200 times greater than the mass of the electron.

[9] Remember, kinetic energy is the energy of motion. If a particle is stationary, kinetic energy is zero. The faster a particle moves, the more kinetic energy it has.

[10] Momentum is the product of mass and velocity, or speed. Bullets hurt because they have high momentum due to their high speed, not because of their relatively small mass. Similarly, if a fast moving bicycle hits a slow moving train, bet on the bike taking more damage, as the train's significantly higher mass gives it far more momentum.

were another particle or two leaving the collision, then these particles could reconcile all of the difficulties. Energy and momentum could both be conserved, as these new particles would carry the balance. Moreover, the electron would then be *required* to have less energy than the muon did, which is consistent with experiment. The great remaining question then is this: why don't the other particles[11] show up in our detectors?

The detectors at the time could only detect particles directly through two forces: electromagnetic forces, and gravity. If the particles in question had no electric charge, then they wouldn't be detectable that way. (Neutrons also show no electric charge.) If their masses were also very small, possibly zero, then we couldn't detect them by gravity. However, if neutrinos exhibit neither of these forces, then a new question arises: what force(s) *do* they experience? Every measureable interaction in nature is driven by some kind of force. We will need more than two forces to explain neutrinos completely.

As it turns out, energy and momentum are conserved in muon decay, as well as all other decay processes. There are, in fact, other particles in play, called neutrinos. Similarly, there is another electron-like particle, called the tau, which is about 3,490 times more massive than the electron, and which may spontaneously decay into neutrinos and either muons or electrons (among other, less common options.)

4.2 Decay Timelines

Radioactive decay is the most common example of the natural creation of particles. This is, then, a reasonable starting point to examine the process. One of the first questions we can ask is "what triggers radioactive decay to produce particles?" We have found that unstable particles can decay when they experience collisions with other particles, or are otherwise given more energy. In the case of individual particles, rather than whole nuclei, becoming a part of a system such as a nucleus can actually increase the stability of a particle.[12] What happens when the particle is isolated, and left to itself?

This is where things start to get truly strange. Left to itself, an unstable particle or nucleus will decay. There is even a defined rate of decay, called the "half life" of the particle. For example, carbon-14 has a half life of approximately 5,730 years. In other words, if you were to take a million carbon-14 nuclei and leave them alone for 5,730 years, then you'd be left with about half a million

[11] Careful studies of these decay processes dictate that not one, but two such exotic particles must be produced in muon decay.

[12] The most common example of this would be the neutron. Though quite stable as a part of most nuclei, a free neutron, left to itself, will eventually decay, transforming into a proton, an electron, and an antineutrino.

carbon-14 nuclei and half a million nitrogen-14 nuclei when you returned. If you left them alone for another 5,730 years, you'd return to approximately a quarter of a million carbon-14 nuclei (and three quarters of a million nitrogen-14.) For each 5,730 years you leave the sample, you'd halve the remaining particle count again. In other words, this is not linear: if you leave for two half-lives, you don't get down to zero, you get down to a quarter of the original.

This has a few implications worth noting.

- The rate of decay does *not* depend on the original particle count.
- The exact decay of a single particle appears to be a random process.

The decay of carbon-14 also has one property worth noting: the total mass of the output particles (nitrogen-14 nucleus, an electron, and an antineutrino) is less than the mass of the carbon-14 nucleus.

We can predict to some accuracy when a given *portion* of a sample will decay. We cannot, however, predict when any specific *particle* will decay. If we had a single carbon-14 nucleus to begin with instead of a million, we couldn't predict exactly when it would decay. There's a 50% chance it would still be carbon after 5,730 years, a 25% chance it would still be carbon after 11,460 years, a 12.5% chance it would still be carbon after 17,190 years, and so forth, but we can never say "this nucleus will decay in exactly four hours and eighteen minutes." If the process were not random, we wouldn't have a definable half life. Either the decay would be triggered by some unrecognized part of the experimental apparatus, in which case two different labs would have two different half lives for the same object, or the decay would be based on some innate "timer" in each particle, meaning we would have a linear decay; if half the sample decayed in 5,730 years, then it would all decay in 11,460 years.

This is the mechanism behind carbon dating; all carbon in nature, including that in living tissue, contains a certain proportion of carbon-14. Living things have mechanisms which will automatically correct errors in proteins and cells if a carbon atom decays into nitrogen. Dead things don't do this as effectively. If radiation detectors determine that half the carbon-14 in an artifact has decayed into nitrogen, then they know the artifact is about 5,730 years old. If they find a different relative proportion, they are working with an artifact or corpse of a different age.

5 What Comes Next

In lesson three, we introduce the Heisenberg Uncertainty Principle, and expand on not only the existence, but on the absolute necessity of the principle for the universe we live in.