Down the Rabbit Hole

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1 Unanswered Questions

1. The basic building blocks of matter, called elementary particles, must all have zero volume. What, then, prevents them from piling in so closely together that the matter they form does *not* have zero volume?
2. Why does the work function kick in so suddenly in the photoelectric effect?
3. If the work function is so well defined, how are electrons arranged within a material?
4. How does electric current really work?
5. How can a particle possibly be some sort of probability field?

2 Welcome To Wonderland

Last lesson, we developed a truly bizarre view of subatomic particles. In this view, they are regions of space corresponding to the *probability* that a particle exists at a given point within that region, and only becomes a single point particle when it interacts with its environment.

The normal human reaction the first time this idea arises is abject rejection. The concept is so far removed from typical human intuition that it is almost irreconcilable for many of us. There are two famous quotes which basically sum up the human reaction to what has come, and to what it still coming:

> Anyone who is not shocked by quantum theory has not understood a single word. *(Niels Bohr, 1885-1962)*

> Baby, you ain't seen nothin' yet. *(Bachman-Turner Overdrive, 1973-present)*

3 The Rabbit Hole Without the Hole

One of the earliest implications of this idea is known as tunneling, and it is one of the fundamental differences between the behaviour we see in the macroscopic world and the microscopic world.

For now, let us look at the macroscopic model. Imagine you are faced with a wall. Your goal is to get the ball in your hand to the other side of the wall. The wall is high enough that you cannot throw the ball over. The wall spreads out to the sides farther than the eye can see, so you cannot go around.

The theories we are used to say that you cannot get the ball to the other side of the wall. This new "region of probability" picture of quantum mechanics says that, if you throw the ball at the wall, there is a small chance that the ball will tunnel directly through the wall, appearing on the other side without harming the wall in any way.

Upon realizing this implication, physicists went charging to their labs, hoping to disprove this insane mental picture of the quantum mechanical world so that it might be replaced with something more sensible.

They failed spectacularly. When you create this situation in the quantum mechanical world, using an electron, photon, and so forth as your ball, you find that some of the balls get through. Moreover, the experimentally determined probability that a ball tunnels through the wall is *exactly* what this insane theory predicts.

Quantum mechanical particles can and do tunnel through seemingly insurmountable obstacles.

A baseball cannot tunnel through a wall very often.[1] It cannot get over because of conservation of energy; if it does not have enough energy to overcome gravity long enough to reach the height of the wall, it simply cannot get through.[2] As we have already seen, quantum mechanical particles can violate energy conservation if they do so in a way that makes the process undetectable until it is too late.

This is exactly how quantum mechanical particles tunnel. When they hit the quantum mechanical version of a wall, they respond to the change in energy levels. The greater the energy required to overcome the obstacle, the less likely

[1] We do not consider walls with windows at this point.

[2] Technically, a baseball is a collection of quantum mechanical particles, and there is definitely a very slim chance of success. The probability of every particle in a baseball tunneling through the wall and coming out the other side looking like a baseball is less than the probability of playing the lottery and winning the jackpot every single week of your adult life.

they are to succeed. As long as they have enough observable energy to exist on the other side of the wall, and as long as we are incapable of observing what happens within the wall while the particles tunnel, they can tunnel through.

There is an even more unexpected flipside to this bizarre phenomenon.

4 Through the Looking Glass. Or Not.

Quantum mechanical particles can tunnel through a wall because of the way they behave at a change in energy. Walls are not the only objects that change required energy.

Let us return to our macroscopic analogy. You are standing on a sidewalk, and you want to throw your ball across the street. In the middle of the street is a sewer, and the manhole cover has been removed. You do not anticipate any trouble throwing your ball across the street, over the empty manhole.

In quantum mechanics theory, the particle reacts to the change in energy here as well.[3] The reaction is even more strange: either edge of the manhole, which both represent changes in energy, can cause the quantum mechanical ball to reflect back upon its original path. In other words, if you throw a quantum mechanical baseball across the street, it reacts to the lip of the manhole as though it hit a wall.

Again, experimentalists thought they had an impossible situation that could be used to derail this insane train of thought. They rushed to their labs and started launching quantum mechanical balls over quantum mechanical holes.

The balls bounced back, from both edges of the holes. They did so just as often as the theory predicted. It was beginning to look like these crazy ideas were here to stay.

[3]Why does an open manhole represent a change in energy? It comes down to something called potential energy, or the energy that can be used to change a system. If you drop a ball from shoulder height to the ground, it picks up a certain speed before it lands. If you drop a ball from shoulder height into an open manhole, it will enter the manhole at the same speed it would have hitting the ground, but then continue to accelerate until it finally reaches the bottom. Thus, when an object is above a hole, it has greater *potential* energy, as it has the potential to reach a greater speed when in free fall.

5 A Window In The Quantum World

There was one other absurdity in this quantum theory that gave classical thinkers hope. These three phenomena came as a set: if you have regions of probability instead of particles, then these regions behave oddly. The region can tunnel through obstacles, reflect at holes, and be in two places at once. If any one of these phenomena could be disproved, then it meant going back to the drawing board and looking for a replacement theory. Any such theory would now need to include tunneling and reflecting, but it was hoped by many that a less insane replacement theory would be found.[4]

Let us go back to our wall. We are still going to try throwing the ball to the other side of the wall, but this time, we are going to install two windows in the wall first. Now it is easy to get the ball to the other side: aim for one window or the other. We will start with open windows to prevent property damage.

With a macroscopic ball, you can observe where it lands, and deduce with certainty which window it came through, particularly if those windows are relatively far apart. Microscopic balls are regions of probability, who spread out widely enough that it is difficult to determine exactly which window the ball went through. To complicate things further, microscopic balls have wave properties within this region, and they interfere with each other as when we discussed diffraction, so if you throw a lot of balls at the windows at once then this diffraction pattern will emerge on the other side. Physicists looked for this, and they found it. To study the phenomenon further, they decided to slow down the rate they threw the balls at the window. If they slowed things down enough, they could guarantee that the balls went through the windows one at a time, and the pattern of interference would be what they expect from a single wave, rather than a combination of waves.

The combination interference pattern stayed. Even when coming in one at a time, it seemed that there were electrons interfering with each other.

The scientists took the next logical step: they closed the windows. If the balls went through a closed window, the window would break, and we would know that a ball had gone through. If the quantum mechanical windows were rigged to repair themselves, then the window breakages could be used to count incoming particles, and they could determine where the extra electrons were leaking into their experiment.

There were no extra electrons. Moreover, as soon as they closed the windows, the pattern of interference instantly became the pattern expected from a single wave, rather than a combination of waves.

[4] We now realize that, in most cases, when a theory fails to describe the quantum world it will likely be replaced by a theory that is *more* insane, not less.

If you do not have a system in place to determine which window the particle goes through, it goes through both, and interferes with itself.

This phenomenon solidified the "region of probability" interpretation of quantum mechanics. There was no other reasonable way to explain observations. It was time to stop fighting the idea, and instead work with the idea to see where it would lead.

6 Electron Orbits

We have already established that the manner in which electrons orbit cannot be exactly the same as planets around a star. The specific and sudden work function from the photoelectric effect was a strong indicator that the energy of current carrying-electrons is in no way random, and that these electrons (at least) are contained in the material with some sort of predetermined structure.

As mentioned earlier, incandescent light bulbs work through blackbody radiation. Fluorescent light bulbs were not mentioned; they do *not* work through blackbody radiation.

Early on, scientists noticed that some gases (such as hydrogen) glow when a current runs through them. This could not be explained with classical physics. Even more confusing, when the glowing light was passed through a prism, it did not produce a continuous rainbow as most light does, but instead produced a series of specific colours. Despite numerous attempts, classical physics failed to describe this phenomenon in any useful way.

Erwin Schrödinger[5] developed a working solution, and he needed this quantum mechanical picture to do it.

In our ball analogy, the potential that determines energy levels is based on gravity. At the quantum mechanical level, the effects of gravity are insignificant. Schrödinger instead developed a model of the atom in which the nucleus is the source of electromagnetic potential, and an orbiting electron is a wave-like region of probability that falls into the potential hole produced by the nucleus.

Trapped in this hole, the electron starts reflecting off of the edges, and interfering with itself inside the span. This interference tends to reduce or eliminate the probability of finding the electron at certain points in this well. In fact, only certain orbits are allowed.

When a wave reflects off a hard surface, part of the wave reverses. The

[5]Erwin Schrödinger: excellent physicist, lousy veterinarian.

wavelength and frequency do not change, but the local shape changes: crests become troughs, and vice versa. Schrödinger found that this can produce a model of electron orbits that explains much of what we have seen. Imagine that exactly half a wavelength of an electron fits in the hole. In that case, the consecutive crest and trough of a single wavelength will reflect back on each other and overlap. However, because the reflection turns troughs into crests and vice versa, the two halves of a single wavelength match (both crests, or both troughs) when they reflect upon each other and overlap in this fashion. They reinforce each other in these orbits, but cancel each other out in others. Similar phenomena happen when the electrons can fit $1\frac{1}{2}$ wavelengths in the well, or $2\frac{1}{2}$, or $3\frac{1}{2}$, or any "whole number plus half" wavelength. Due to difficulty in the math,[6] Schrödinger could only solve the problem for a single electron atom, which means hydrogen, or helium missing one electron, or lithium missing two electrons, or ununhexium missing 115 electrons, and so forth.

The wavelength of an electron depends on its energy. That means that there are preferred energies for electrons to have while they are in these orbits. When calculating the differences between the energies of these different allowed orbits, Schrödinger realized that they are exactly the right differences in energies to match the energy of light emitted when a current runs through hydrogen.

The idea that electrons exist as regions of probability with wave-like properties not only predicted some bizarre behaviours as described earlier, but it perfectly explained the spectra of light emitted from a gas subjected to electric current. The current imparts orbiting electrons with enough energy to jump up to higher orbits. Electrons then "fall" to lower orbits, emitting energy in the form of a photon of light. That photon then contributes to the glow of the light.

This is a huge building block for many of our unanswered questions. Electrons are not only allowed to exist in orbits that line up with the self-interference described above, but these are the only orbits they are allowed to use. Instead of orbiting a nucleus like little planets, they extend through a region like a wave, and these regions come in some very bizarre shapes that will be discussed in our next lesson.

[6] This is not to say Schrödinger was a poor mathematician. The math is what we call a "transcendental equation." This means that, if the variable you are interested is x, then there is no possible way to manipulate the equation to turn it into $x = \textit{stuff that doesn't involve } x$. It's not just that he couldn't do it, but that nobody could do it. Instead, we solve equations of this type using highly sophisticate "guess and test" methods, where each successive guess leads to a better guess until our final guess becomes close enough for the purpose at hand.