Like a Record, Baby

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1 Unanswered Question

1. Why do particles act like bar magnets?
2. Why does Pauli's Exclusion Principle exist in nature? If these particles have zero volume, why *can't* two electrons share the same orbit?

2 Angular Momentum

Before we get back to answering our questions about quantum mechanics, we need to lay out an important part of classical mechanics.

Classical mechanics involves a quantity known as "angular momentum," which is conserved. Most people will have an understanding of what those two words mean in isolation, but the combination is something of a mystery.

Imagine a figure skater on an ice rink. This skater starts to spin on the spot, with her arms out.[1] She then folds her arms in, and as a result, starts to spin faster. When she stretches her arms out again, she slows down again. No outside forces are acting on her, and she isn't pushing along the ice; she can do the same thing whether on the ice or in the air. How does that work?

This works because the quantity known as "angular momentum" is conserved. This is a combination of two factors:

1. Position: where an object is, and
2. Momentum: how quickly an object with inertia is moving

[1] The logic holds true for male skaters, too, but when the author can imagine any person at all wearing a skintight body stocking in a cold environment, the author imagines a shapely female.

The rest of this section gets into heavy mathematical logic, although the actual algebra is minimal. If you are satisfied knowing only that angular momentum depends on position and momentum, and that if either position or momentum is zero then the angular momentum is zero, then feel free to skip ahead to the next section if the going gets rough.

The way position and momentum combine to form angular momentum depends on both the magnitude of these quantities, and their directions, and is most obvious when the motion is circular. If the object in motion is tied to a pole, or a spinning ice skater, then it is forced to move in a circle. Let us choose the skater's right hand as the object of interest. Assume that the ice skater is rotating around a line that goes through her exactly down her middle. Assume also that, when viewed from a camera directly above her head, she is spinning counter clockwise.

If her arm is completely outstretched, then the magnitude of its position is half her shoulder width plus the length of her arm, as we measure from the skater's centre, which she rotates around. The direction from her centre to her hand is directly along her arm.

The momentum of her hand will have a given magnitude according to how rapidly she is currently spinning. The direction is the tricky part. Picture the skater in a moment when she is facing due North. At this moment, her right hand is extended due East. Because we decided she is spinning counterclockwise when viewed from above, her right hand is traveling towards the North at this instant. In fact, the position of her hand is always to her right, and the direction of travel is always the direction she is facing, perpendicular to her outstretched arm.

The angular momentum when position and momentum are perpendicular is merely the product of the two quantities. So, if the position of her hand is $1m$ to her right,[2] and the momentum of her hand is 1 in the appropriate units, then her angular momentum is also 1.[3]

Now imagine she pulls her arms in, and the magnitude of the position of her right hand becomes a quarter of what it used to be. Angular momentum is subject to conservation: if we cut the position by four, the momentum must be multiplied by four. This is why the skater speeds up in her spin when she pulls her arms in; the product of the positions of her hands, arms, sides, and other body parts with their respective momenta must be constant. If her body parts get closer to the centre, the positions get smaller in magnitude, and the skater

[2] She's a gangly one, our figure skater.

[3] The angular momentum has a direction, too, although that won't be terribly important for our purposes. In this case, the direction is up. Take your right hand, open it flat, point your thumb in the direction of her hand's position, and point your fingers in the direction of her hand's motion: your palm now points in the direction of the angular momentum.

speeds up.

3 Spin

Any time a charged particle moves, it produces a magnetic field. All magnetic fields carry angular momentum. The reasons for these phenomena will not be dealt with here.[4]

What we have with electrons, protons, neutrons, and their component particles are electrically charged particles that exist in regions of probability. When these regions overlap, they interfere with each other, and the manner in which they interfere and interact depends on the fact that each of these particles acts like a little bar magnet.

These particles act like bar magnets because they carry angular momentum. We call this angular momentum *spin*. Even more surprising, this angular momentum is a fundamental property of the particles. Every electron carries exactly the same amount of angular momentum, which is the same amount carried by a proton, the same by a neutron, and the same by the muons, taus and neutrinos mentioned several lessons back. Every photon carries the same angular momentum as every other photon, although the angular momentum of a photon is different from that of electrons, protons, neutrons and the rest. Other particles, such as the W, Z, graviton and gluon particles also carry angular momentum, or spin.

What are these particles rotating around? We don't know. Are they even rotating at all? We don't know. If they are rotating, we would need to determine what they are rotating around, and where the momentum came from in the first place. We would also need to explain why every electron carries angular momentum *identical* to that of every other electron, regardless of the situation. If we explore the behaviours and properties that particles have because of this spin, then perhaps we can sort out what spin really is.

[4] Before Einstein, the first phenomenon was jammed into the theory quite forcefully because the behaviour was noted in the lab. Once that had been established, the second followed, not by logic, but "because the math says so." After Einstein, both can be explained through the theory of relativity with charged particles, but that is well beyond the scope of this series. Perhaps relativity will appear in some other year's summer school curriculum, should demand warrant another series.

4 Fermions and Bosons

The spin particles have come in two different varieties, which lead to two completely different sets of behaviour. We can, therefore, classify all particles by these behaviours. The categories are:

- **Fermions** - These particles are named after Enrico Fermi, who (with the help of Paul Dirac) was able to describe their behaviour. The fundamental point of the behaviour is that these particles obey the Pauli Exclusion Principle, as described last lesson. Electrons, muons, taus, neutrinos, protons, neutrons, and the quarks protons and neutrons are made out of fall into this category.
- **Bosons** - These particles are named after Satyendra Nath Bose, who (with the help of Albert Einstein) was able to describe their behaviour. They are *not* subject to the Pauli Exclusion Principle, and can coexist with others of their kind without a problem. Photons, gluons, W, Z, gravitons, and any other force carrying particles fall into this category.

These are two fundamentally different behaviours. Now, the problem arises when considering the differences between them. Fermions and bosons are all zero volume particles. Why would they possibly be limited in their ability to share space? The thought makes sense if the particles have volume, but they do not.

We do know this: the behaviour in question is a fundamental part of the universe we live in. If the Pauli Exclusion Principle didn't apply, it would be virtually impossible to form molecules with anything but hydrogen, as the electrons could all pile into a single orbit. Clearly, some property of these particles keeps them distinct and unable to share the same space. It is easy to imagine that the interactions of the fields themselves are what keeps them from sharing. However, that leads to another problem.

Bosons are made up of fields, just as fermions are. Yet, bosons are not subject to the Pauli Exclusion Principle. If they were, they would be unable to mediate forces as they do, because the number of virtual particles that could be exchanged would be limited. If they were limited, we would not see forces as the continuous actions they seem to be in the macroscopic world, and they would not be stable enough to hold our observable world together. In short, the universe we live in needs bosons.

Even more strange is the way they combine. In day to day life, we can put puzzle pieces together to make a picture, but we cannot put pictures together to make puzzle pieces. This is similar in the cases of fermions and bosons. An even number of fermions can combine to make one large boson, but no quantity of bosons can combine to make a fermion.

Again, this is strange, but necessary for what we see. Superconductivity works this way; the conducting electrons join up in pairs, and these pairs act like bosons. They can then all share a single, defect free orbit allowing them to traverse the entire solid with no electrical resistance at all.

Why do we need an even number of fermions to make a boson? It is nothing more than adding fractions. Every spin for a particle can be represented as a multiple of a constant called $\hbar$.[5] Bosons are whole multiples of this number, whether that multiple is 0, $\hbar$, $2\hbar$, $3\hbar$, etc. Fermions, on the other hand, are half multiples of $\hbar$, and can be $\frac{\hbar}{2}$, $\frac{3\hbar}{2}$, $\frac{5\hbar}{2}$, $\frac{7\hbar}{2}$, etc. When two or more particles bond themselves together, these spins can add or subtract (according to the bonds). If you combine an even number of fermions, adding or subtracting their spins gives you an even multiple of $\frac{\hbar}{2}$, which can then be reduced to a whole number multiple of $\hbar$. If you have an odd number of fermions, you are back to having an odd multiple of $\frac{\hbar}{2}$, and the combination is a fermion again. However, no amount of manipulation of boson spins through addition or subtraction will make fractions appear; bosons can only combine to form bosons.

5 Spin and Behaviour

This leaves a single, burning question:[6] *why* and *how* can a quantity like angular momentum determine whether or not particles can share orbits?

When Wolfgang Pauli first presented his reasoning for why the spin and behaviour of particles should be linked, he essentially wrote a seven page paper that said "because the math says so." Many researchers were hoping for a conceptual reason for this connection. After all, math should not dictate a scientific theory. It can point us in directions to explore, and it can make it possible to use quantifiable experiments to verify theories; math is undoubtedly an indispensable asset to the scientist, and its critical importance cannot be underestimated. However, our mathematical models should be consistent *representations* of the physical models, and not the models themselves.

To that end, Raymond F. Streater and Arthur S. Wightman rewrote the argument in *PCT, Spin and Statistics, and All That.*[7] It is a 199 page volume

[5] Do not worry about the notation of this constant: it is, in essence, a really small number with units of angular momentum.

[6] Actually, there are still a LOT of burning questions in quantum physics, but this is the last one the author is going to point out in this series.

[7] ISBN 0-691-07062-8

that says, in essence, that angular momentum and behaviour are connected "because the math says so."

Not satisfied with this, Ian Duck and E. C. G. Sudarshan endeavored to expand this further. In *Pauli and the Spin-Statistic Theorem*[8], they strive to explain this connection at an elementary level. Their volume is 503 pages of "because the math says so."

Richard Feynman was famous for his ability to teach physics at any level to anyone. He maintained that, if you cannot explain a concept to a stranger in the time it takes to ride an elevator, then you do not truly understand that concept. Here is what Feynman, master teacher, had to say about this connection:

> Why is it that particles with half-integral spin are Fermi particles ... whereas particles with integral spin are Bose particles...? We apologize for the fact that we cannot give you an elementary explanation. An explanation has been worked out by Pauli for complicated arguments of quantum field theory and relativity. He has shown that the two must necessarily go together, but we have not been able to find a way of reproducing his arguments on an elementary level.... This probably means that we do not have a complete understanding of the fundamental principle involved....

In short, this is not a question that can be answered without mathematics at this time. The mathematical answer is called the "spin-statistics theorem," and can be found in the aforementioned resources. At the moment, the exact nature of spin and its connection(s) to the way particles behave is unknown. All we know for certain is that both types of particles exist, and a quantum physics theory that is consistent with Einstein's relativity requires them to exist. The specific reasons why are still a mystery.

6 Conclusion

Our knowledge and understanding of our world has increased dramatically over the last century or so, but we're still a long way from finished. Still, the concepts presented in these past nine lessons are those which have stood up to repeated and dedicated attempts to disprove them. That's not a complete guarantee that they are correct, but the power of science lies in its ability to place limits on how wrong we are. Any theory that eventually replaces these will show virtually identical behaviours under the laboratory environments that we have produced

[8]ISBN-10 981-02-3114-8, ISBN-13 978-981-02-3114-9

to date. However, traditionally speaking, every upgrade which expands our laboratory conditions has lead to wholly unexpected behaviour. With the LHC now operational, it's only a matter of time before the more tenuous ideas developed by man are either confirmed our outright replaced.

7 Afterward

This concludes the first summer school session at Bureau 42. If you send an e-mail to `fiziko@bureau42.com` with "Completion Certificate" in the subject line and your name in the body, then a certificate of completion will be e-mailed to you in PDF format within 90 days.