Math From Scratch Lesson 33: Constants, Parameters and Variables

W. Blaine Dowler

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1 Intentions

Up to this point, we have focused on a number of elements required for algebra, and have described the 13 axioms which define the real numbers. Our next goal will be to extend the applications of these axioms into real world situations, and use this to motivate further definitions of mathematical objects and goals.

2 Algebraic Equations and Their Components

We have run into numerous algebraic equations thus far, often used as examples. These are the core of much of science. Fields such as physics and chemistry are founded on the principle that all observable phenomena can be described with mathematical models. One example of such a model is that used to describe the distance d traveled by an object with an initial speed v and constant acceleration a over the amount of time t. That equation is

$$d = vt + \frac{1}{2}at^2$$

where we have used the convention that adjacent variables are to be multiplied together. This equation has one *constant*, two *parameters*, and two *variables*.

2.1 Constants

A constant is something that takes on the same value[1] in every application of the model, regardless of the individual problem at hand. In the above example, $\frac{1}{2}$ is the constant, since it will be $\frac{1}{2}$ every time this model is correctly applied regardless of the current situation under investigation.

2.2 Parameters

A parameter is something that varies from problem to problem, but remains constant for the duration of a single problem. In this case, v and a are parameters. If you drop a ball of the roof of a building on planet Earth and wish to track its descent, you would use $v = 0\frac{m}{s}$ and $a = 9.81\frac{m}{s^2}$ for every time unit t you which to examine. (i.e. regardless of whether you wonder where the ball is after 1 second, 2 seconds, 3 seconds, etc., you will use $v = 0\frac{m}{s}$ and $a = -9.81\frac{m}{s^2}$ when doing your calculations.) If, instead, you wish to examine the position of a car coming to a halt before reaching a red light, you might use $v = 30\frac{km}{h} = 8.3\frac{m}{s}$ and $a = -25\frac{m}{s^2}$. The values of v and a change in different situations, but they do not change in the course of a single physical event.

2.3 Variables

A variable is something that can take on multiple values in the course of a single examination. In this case, d and t are both variables. For each t, there is one, and only one, value of d for which the two sides of the equation are mathematically equal. For each d, there may be one or two values of t which

[1]Note that the use of the "same" value is limited to units and accuracy. If we use the speed of light in a vacuum, it may be treated as 186000 miles per second, 299792458 meters per second, or 1.802×10^{11} furlongs per fortnight, but it's still the speed of light in a vacuum to some degree of accuracy.

apply,[2] but you can choose either d or t in an arbitrary fashion and work with the model.

3 Conventions

With these different types of algebraic objects, it is most useful to assign conventions to their labels. Thankfully, the mathematical community has developed just such a set of conventions.

Constants Constants are generally expressed in their exact values, such as 2 or 3. If the constants are not rational numbers, or some other easily expressible number, then they are typically assigned lowercase letters from the Greek alphabet, such as π.

Parameters Parameters are generally assigned lowercase letters from the first half of the Latin alphabet, running from $a - m$. This may be the reason that the slope of a straight line is given by m rather than s, although the exact origin of that label is not clear from history.[3]

Variables Variables are generally assigned lowercase letters from the latter half of the Latin alphabet, running from $n - z$.

Capital letters from the various alphabets are customarily assigned to quantities of types we have yet to discuss. Of course, these conventions are really rules of thumb that apply only to pure math; when looking at applications to science or other "real world" quantities, the first letter of the term in question is often used regardless of the position within the alphabet. For example, the variable d represented "distance" or "displacement" in the equation

$$d = vt + \frac{1}{2}at^2$$

discussed above.

[2]The means to count how many values of d and t there are under different circumstances will be described in a later lesson. Since this is merely an example and not a definition, we have maintained our goal of keeping things in prerequisite order.

[3]There are also unsupported theories that it came from the French word "monter" for "to climb," but the letter predates the use of the term "slope" in any language. The quantity was labeled with an m when the slope was still referred to as "the tangent of the line to the x-axis," so the translation of a term that wasn't even in use seems unlikely.

4 Next Lesson

In our next lesson, we apply these vital definitions to real world examples of algebraic equations, and begin to look at proper techniques for isolating variables and parameters within these equations.