Quanta, Quanta Everywhere

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1 Unanswered Questions

1. The basic building blocks of matter, called elementary particles, must all have zero volume. What, then, prevents them from piling in so closely together that the matter they form does *not* have zero volume?
2. How can we reconcile the Poisson spot with the photoelectric effect and blackbody radiation?
3. Why does the work function kick in so suddenly in the photoelectric effect?
4. If the work function is so well defined, how are electrons arranged within a material?
5. How does electric current really work?
6. How do we know the Heisenberg Uncertainty Principle applies to particle existence, and not merely measurement?
7. How can particles with zero volume interact through virtual particle exchange at all?

In this lesson, we focus on questions #2 and #6, and the implications of their solutions.

From question #2, we have contradictory results to reconcile. There is no question that light behaves as both a particle and a wave. This was a revelation. The first step in the investigation of this problem was to discover if light was the only object in nature to suffer from this problem.

2 Diffraction

To investigate whether or not other particles behave like waves, we must first establish some properties and behaviours that are unique to waves.

The first such behaviour is diffraction. As mentioned in the previous lesson, waves can interfere with each other. In the case of light, that interference produces bright and dark spots in a regular pattern, depending only upon the geometry of the situation and the wavelength of the light.[1] The setup requires a point source of light, a screen to observe the light, and some sort of physical block or impediment that ensures light falling on the screen has as least two discrete paths to follow to arrive at the final destination. This can be done by putting an opaque screen with two slits between the light source and the screen. Alternatively, it can also be done by setting up two mirrors with less than 100% reflection, allowing light passing through to reflect off of either mirror.[2]

Imagine that a wave of light passes through the longer path, and arrives at the screen when its wave is at a crest, or the highest point. Now imagine a second light wave travels the shorter path, and arrives at the screen at exactly the same time as the first wave of light. If that second wave is also at a crest, then the two will combine to make a bright spot. If that second wave arrives at a trough, on the other hand, the two will cancel each other out, producing a dark spot. This not only shows the wave behaviour of light, but it means that an analysis of the complete pattern resulting can determine the wavelength of the light involved. (Bright spots appear when the difference in distance traveled by the two waves along two paths is a whole number of wavelengths. Dark spots appear when the difference is half a wavelength more than a whole number of wavelengths.)

In 1927, Lester Germer and Clinton Davisson set up an experiment similar to the mirror setup described above. Instead of mirrors, they used nickel, which reflects electrons. Instead of a light source, they used a source of electrons. Instead of a screen, they used a device that counts electrons.

They found places where they counted a lot of electrons, and they found places where they counted very few electrons. In short, electrons combine with each other like waves. Instead of being brighter or darker, there are more and less of them.[3]

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[1] The light used for this must be of a single colour, or a single wavelength. White light and so forth can get too muddled to see the effects clearly in many cases.

[2] These are certainly not the only arrangements, but they are among the most popular and easiest to set up.

[3] By this stage, it probably comes as no surprise to learn that "eureka!" is almost never spoken aloud in physics research institutions. However, it is very common to hear variations of "what the *expletive* is *that*?"

This is staggering. It has also been reproduced for protons, neutrons, and any other particle we have examined this way.

The wave nature of the objects we are used to calling particles is not only present, but it seems to impact whether or not the particles even exist. The number of electrons launched at the nickel, the total charge on them, and their total kinetic energy are all observable quantities, so they must conform to conservation laws. The electrons have to go *somewhere*, and they do. Somehow, some way, these little zero volume billiard balls we have been picturing move through space as though they have already chosen their destinations based on where other particles are in the same situation.

This gets into an information exchange problem. Again, the particles seem to demonstrate precognition, arriving where they need to arrive to coincide with other particles. We have already developed a means of information exchange between particles, but it ties directly to forces. The forces involved with these particles are not consistent. We know that electrons repel other electrons, and yet they are clumping together at the same destination. We know that the only long distance force neutrons experience is gravity, and that force is not nearly strong enough to account for this behaviour. No, the particles cannot be exchanging information. There must be another answer.

3 Particle or Wave?

The first attempt to reconcile these ideas was something called "wave-particle duality." Scientists realized that there was no way to treat subatomic objects as either particles or waves. These entities exhibit properties of both. The solution was to propose that they are both. Every basic building block in the Universe is both a particle and a wave, exhibiting all properties of both when placed in situations where those properties matter.

As hard as this was to conceptualize, it did provide the logical consistency required for scientists to accept things as they saw them. This explains much of what we see, but not all. It explains why light behaves like a particle, and also why electrons have the wavelength necessary to demonstrate diffraction. It does not, however, effectively deal with the problem of a precognitive electron waving to exactly where it needs to be: the number of electrons we count at the destination still varies as though the electrons already know where to go.

3.1 None of the Above

The proper solution to the problem is even more difficult to grasp. Electrons, photons, protons, neutrons and all of their ilk are neither particles nor waves, nor are they any combination of the two. They are something else, something entirely different, which has no macroscopic analog that we can relate them to from our day to day experience. We need to develop an entirely new mental picture to understand what these entities truly are. To that end, as promised in lesson three, it is now time for Heisenberg to make his triumphant return.

4 Heisenberg and Existence

If the electrons, protons, and other particles that exhibit wave behaviour determine their destinations before arriving at them, then we find ourselves stuck with precognition and forceless information exchange. However, they certainly react to other particles traveling similar journeys; we know this because they behave like waves.

The solution to this seeming discrepancy is as bizarre as the problem itself: the determination of where particles "arrive" at the detector isn't made until they get there. Along the way, these particles are not the zero volume particles we have deduced them to be. Rather, they are regions in which it is probable to find said zero volume particle when you look for it.

These regions of probability are the ones that demonstrate the wave behaviour. When we put the electron counter in place, and force an interaction that depends upon the precise current location of the particle, then the region "collapses" to a single point which represents our zero volume particle. There is no time required for this collapse, either, so the constraints on the wave image of information transmission from lesson four do not apply. Instead, the Universe behaves as though the particle had always been exactly where the region collapsed in the first place.

When calculations were done for this model,[4] they demonstrated that the indeterminate regions follow identical limitations to the Heisenberg Uncertainty Principle. This "fuzziness" we need in the existence of our particles exactly matches the fuzziness in our ability to measure and detect them. This is the

[4] The calculations were done with the intent of demonstrating the utter ridiculousness of the model so that it could be thrown out and replaced with something sensible. This was common. It was also common to perform experiments to prove the utter ridiculousness of these theories, only to find the rules that govern the Universe are at least as ridiculous as mankind can imagine. More on this next lesson.

proof we were waiting for to justify the statement that the Heisenberg Uncertainty Principle applies to existence as well as measurement.

This is how we answer the last unanswered question on this lesson's list: zero volume particles can interact with each other because the virtual particles they exchange do not need to reach a zero volume particle, but instead need to reach a region of space with more than zero volume. When two such fields overlap, then there is a probability of interaction, and this is the interaction that is perceived as a force.

5 Inverse Square Laws

Now that we have established that zero volume particles still have physical extents when not being observed, we can establish another important characteristic of the electromagnetic, gravitational, and weak nuclear forces. This section is more mathematically involved than most, and is meant to explain a phenomenon to people familiar with the particulars of a force. The most important point here is that the force between two charged objects changes more rapidly if we change the distance between them than if we change the charges on them. Readers who are not mathematically inclined can ignore the rest of this section without impacting the readability of the remaining lessons.

The electromagnetic and gravitational forces both have the same basic algebraic structure:

$$F = \frac{cp_1p_2}{r^2}$$

In this notation, c is a constant that is representative of the strength of the force in general,[5] p_1 and p_2 are properties representing the specific strength of the force for the two objects experiencing the force,[6] and r is the distance between the two objects.

The question is this: why do the forces always take this particular form? The properties in the numerator match intuition: if you double the property that generates the force, perhaps by doubling the mass of one object, then you double the strength of the force. However, the less intuitive part is the exponent 2 associated with the distance between them. If you have two masses a certain distance apart, double the mass of one and then double the distance between them, our intuition would often tell us to expect the force to remain the same.

[5] $c = G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$ for gravity, $c = k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{Nm^2}{C^2}$ for electromagnetism

[6] $p = m$ for mass or inertia with the gravitational force, $p = q$ for electric charge for the electromagnetic force

Instead, the force between them is cut by half. We refer to this as an *inverse square law*: the force depends on the inverse square of the distance.[7]

5.1 Classical View

In the classical view, the exponent of r turns out to be rather easy to explain, provided you can imagine a sphere. In this view, each object produced a field. For example, let us imagine the two objects are the Earth and the Moon, and that the force between them is gravity. Earth would produce a gravitational field, which would be perfectly uniform in all directions if Earth were a perfect sphere.[8] The field would then be distributed through space evenly in all directions. As you move farther from the Earth, the field would be spread out more thinly.

Now let us imagine the Moon being placed in this field a distance r away from the Earth. The Moon is roughly spherical: when viewed from the Earth, it looks very much like a circle. The strength of the force it will experience is then going to depend on the proportion of Earth's gravitational field that it experiences. If the circular face of the Moon has an area A_{Moon}, then the strength of the force it experiences will depend on the percentage of the Earth's gravitational field that it inhabits. This percentage is determined by the ratio of the Moon's area to the total area in space that the Earth's field is spread over (A_{field}), which (mathematically speaking) is this ratio:

$$\frac{A_{Moon}}{A_{field}} = \frac{A_{Moon}}{4\pi r^2}$$

where we have used the fact that the area of a sphere of radius r, which the Earth's field is spread over, is given by $4\pi r^2$.

This is where the r^2 dependence comes from in the classical view. The geometry that deals with the 4π and other components gets absorbed mathematically in the properties (m or q) of the charges and the constant (c) in the force equation.

This view is based on the classical notion that we can model the field of one object in isolation, and then add a smaller object to the system without significantly impacting the field produced. In our quantum mechanical view, we

[7] The "inverse" part means r appears in the denominator, or bottom, of the fraction instead of the top, while the "square" part refers to the exponent 2.

[8] Earth actually bulges at the equator due to its rotation. This also means that you will lose a bit of weight if you move closer to the equator, without changing your mass, simply because of this same r^2 dependence in the denominator of our force equation. The average distance between yourself and the centre of the Earth would increase closer to the equator. You will lose more weight if you walk or run to the equator in the process.

understand that forces only exist when you have two entities involved, which can both send *and receive* virtual particles. Thus, you cannot create a field with an isolated particle when no interactions are possible, and the virtual particles that mediate the exchange do not distribute themselves in a perfect sphere, but rather appear in random directions and only matter when they interact with other particles. We need a new mental picture to explain the inverse square forces.

5.2 The Quantum View

As a first attempt at building a view of inverse square laws with our quantum view, we could attempt to say that the extent of our region of probability, or field of influence, acts like the visible area of the Moon acted in the classical view. This seems acceptable at first, but fails when we realize the virtual particles are not spherically distributed unless the particles they can react to are spherically distributed.

To explain things at the quantum level, we need another picture. The forces only exist when the objects exist in pairs, so we cannot model one field and then insert a second object into it.

Recall that the virtual particles exist within the limits of Heisenberg Uncertainty, which (as we can now confirm) limits existence as well as measurement. Let us also recall that the energy transferred by these virtual particles only "counts" if the particles reach a destination, at which time they are observable and subject to conservation of energy.

The particles of energy E that reach their destination within time t are then constrained by $Et < k$, where k is some constant about the situation we do not yet know. (It will vary from any given situation to any other, and cannot be exactly determined for a general discussion.) Keep in mind, also, that the Universe enforces a speed limit of the speed of light in vacuum. That means there is a limit to how far a particle can travel in time t. If we exchange high energy particles, then they can only be exchanged over short distances, because of this speed limit. If we exchange low energy particles, then they can be exchanged over larger distances, because a longer transit time is allowed by Heisenberg.

This is the dependence we need. The virtual particles contributed to the force by one real body will fade smoothly as the distance is increased. If the energy of the virtual particles produced is directly related to the charge property (p) of the real particle, then each real particle contributes a factor of $\frac{p}{r}$ to the overall force. In this case, the force equation for our quantum mechanical view becomes

$$F = c\left(\frac{p_1}{r}\right)\left(\frac{p_2}{r}\right) = \frac{cp_1p_2}{r^2}$$

because the distance r between the real particles is identical from each particle's perspective.

This is exactly the same algebra that was produced by the classical view that is consistent with experimentation, which is what we needed. We can then explain the forces experienced by large objects, like the Earth and the Moon, by adding up the influences of every individual particle involved. Thankfully, the calculation involved in the sum is exactly the same as doing a single calculation looking at the total charges between the two summed over all particles involved in both bodies, using the distance between their centres as the total distance between them, which is exactly the classical formula.[9]

[9] The alternative would be to perform over a googol's worth of calculations between each particle in the Earth and each particle in the Moon and add up the results.