Alternative Interpretation of $PV = nRT$

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Abstract

This tidbit reviews the Ideal Gas Law and looks at alternative ways to interpret the variables and relationships involved. The approach comes through statistical physics, via a result not generally introduced at the high school level.

1 The Ideal Gas Law

Those who have taken high school chemistry typically are (or once were) familiar with the Ideal Gas Law and the variables it uses. For review, the law is

$$PV = nRT$$

where P is the pressure in the gas, V is the volume the gas occupies, n is the number of moles[1] of gas we have, $R = 8.31451\frac{J}{K \cdot mol}$ is the ideal gas constant determined by experiment, and T is the temperature of the gas in degrees Kelvin.

To apply the Ideal Gas Law, we need a special type of gas. In an ideal gas, the molecules of the gas do not interact with each other in any way, so that the behaviour of the gas becomes entirely independent of the particle density. While there are no actual ideal gases in nature, some gases behave in manners similar enough to ideal behaviour that the mathematical structure around an ideal gas is still a useful construct. (The noble gases, on the far right of the periodic table, fit this model relatively well. Other gases do not fit this model as well.)

2 Boltzmann and Statistical Physics

Statistical physics is the field of physics that deals with the average behaviours of particles in given situations. It is this field which found an interpretation of temperature.

[1]Remember, a "mole" is like a fancy dozen that is convenient for chemistry. A dozen Hydrogen atoms means 12 Hydrogen atoms, while a mole of Hydrogen atoms is 6.022×10^{23} Hydrogen atoms. It's an oddball number because it was defined as the number of molecules of Hydrogen gas in a container of convenient volume at convenient temperature and pressure; the actual number was calculated after it was defined.

All particles have some amount of random motion involved in their behaviour. The greater the energy of motion, or kinetic energy, a particle contains, the more random motion occurs. When particles like this vibrate and interaction with other particles, they collide, and transfer some of this energy. On large scales, we perceive this energy as heat energy, and it is measured as temperature. The relationship that connects temperature as we measure it to the kinetic energy of an individual particle is

$$E_k = k_B T$$

where E_k is the kinetic energy of the particle, T is the temperature, and $k_B = 1.3807 \times 10^{-23} \frac{J}{K}$ is the Boltzmann constant, determined by experiment.

3 A New Interpretation

Imagine we take our single particle using the physics approach, and apply $E_k = k_B T$. Now, what if we have two particles, which do not interact with each other? The total energy for the two of them would be given by $E_k = 2k_B T$. If we had three particles, we would use $E_k = 3k_B T$. If we had an entire mole of these particles, we would have $E_k = 6.022 \times 10^{23} k_B T$. Here's the part where the new interpretation comes in: $6.022 \times 10^{23} \times 1.3807 \times 10^{-23} \frac{J}{K} = 8.31 \frac{J}{K} = R$.

In other words, the nRT side of the Ideal Gas Law represents the total thermal kinetic energy contained in the n moles of gas contained in our sample.

Because of the equality in the Ideal Gas Law, the left hand side, PV, must also be equal to the total thermal kinetic energy of the gas sample. This can lead to a new interpretation of pressure, as well. If $PV = E_k$, then $P = \frac{E_k}{V}$, meaning the pressure we perceive from a gas can be interpreted as the average amount of thermal kinetic energy per unit volume in the gas. The more kinetic energy the particles have, the more they vibrate randomly, and the more frequently they collide with the sides of the container they are in; this is what we perceive as pressure.

In short, this is one way in which different sciences cross over with each other in ways that can add to our understanding and interpretation of the world around us. Each side of $PV = nRT$ can be used to represent the kinetic energy of the particles in the gas, and pressure is the average amount of kinetic energy per unit volume.